

Nonlocality and entanglement as opposite properties

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We show that, for any chained Bell inequality with any number of settings, nonlocality and entanglement are not only essentially different properties but opposite ones. We first show that, in the absence of noise, the threshold detection efficiency for a loophole-free Bell test increases with the degree of entanglement, so that the closer the quantum states are to product states, the harder it is to reproduce the quantum predictions with local models. In the presence of white noise, we show that nonlocality and entanglement are simultaneously maximized only in the presence of extreme noise; in any other case, the lowest threshold detection efficiency is obtained by reducing the entanglement.

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Introduction.—Nonlocality and entanglement are two core concepts in quantum information. If $p_\rho(ab)$ is the joint probability that Alice obtains $a = 1$ and Bob $b = 1$ on a system prepared in state ρ , nonlocality is the impossibility of expressing $p_\rho(ab)$ as $\sum_\lambda p_\rho(\lambda)p_\rho(a, \lambda)p_\rho(b, \lambda)$, where λ are preestablished classical correlations [1]. Entanglement is the impossibility of writing down a quantum state as a convex combination of separable states. Nonlocality and entanglement are related concepts in the sense that, to have nonlocality, entanglement is needed [2]. The difference between both concepts has been pointed out before: first, noticing that there are entangled states which do not violate specific Bell inequalities [3], and then pointing out that states with lower entanglement lead to larger violations of some specific Bell inequalities [4–7]. The difficulty in reaching a general conclusion about their relationship is that of finding a general scenario where incontrovertible measures of nonlocality and entanglement can be compared.

A bipartite scenario has the advantage that any of the many measures of entanglement assigns zero entanglement to product states and maximum entanglement to maximally entangled states [8]. Nonlocality is a more delicate issue in that different restrictions on the number of settings usually lead to different measures of nonlocality. This suggests that any conclusion should be based on a bipartite scenario in which the parties can perform an arbitrary number of measurements.

The aim of this Letter is to show that, in such scenario, and assuming some natural measures, nonlocality and entanglement actually have opposite behaviors.

In [9, 10], Braunstein and Caves (BC) introduced a generalization of the Clauser-Horne-Shimony-Holt (CHSH) [11] and Clauser-Horne (CH) [12] Bell inequalities, known as chained Bell inequalities, in which Alice and Bob choose among $M \geq 2$ settings. Chained Bell inequalities have some interesting applications: case $M = 3$ fixes a loophole that occurs in some experiments based on the CHSH inequality [13]. Besides, it reduces the number of trials needed to rule out local hidden variable theories [14], and improves the security of some quantum key distribution protocols [15]. In the case in which M tends to infinity, the inequality allows one to discard nonlocal hidden variable theories with a nonzero local fraction [16]. Chained Bell inequalities have been experimentally tested using pairs of photons, with $M = 3$ [17], 4 [18], and 21 [19].

A natural measure of nonlocality is the minimum detection efficiency required for a loophole-free violation [20]. The idea is simple: when one observes a violation of a Bell inequality with perfect detection efficiency, this means that no local model can reproduce the joint probabilities. If the critical detection efficiency is η_{crit} , this means that no local model exists, even if one locally rejects a fraction $1 - \eta_{\text{crit}}$ of the events. Therefore, the smaller η_{crit} , the harder it is to reproduce the joint probabilities with local models; the smaller η_{crit} , the larger nonlocality.

The minimum detection efficiency required for a loophole-free violation of chained Bell inequalities for any $M \geq 2$ using maximally entangled states has been obtained in [21]. The fact that the maximum quantum violation of chained Bell inequalities is always achieved with

maximally entangled states [22] might suggest that the minimum detection efficiency occurs for maximally entangled states, but no proof exists of whether the detection efficiency for the chained Bell inequalities can indeed be reduced when one considers more general classes of entangled states. Indeed, for case $M = 2$, corresponding to the CH inequality, the minimum detection efficiency occurs for almost product states [23, 24]. In the following, we will show that, in the absence of noise (e.g., considering pure states), the minimum detection efficiency for any chained Bell inequality is obtained for *almost product states* for any M . Additionally, we will show that, if the state is affected by a (small) amount of white noise, then the largest nonlocality (i.e., the lowest detection efficiency) requires only a (small) amount of entanglement.

Detection efficiency for chained Bell inequalities.—The version of the chained Bell inequalities introduced in [18], which is symmetric under the permutation of Alice and Bob, reads

$$(S_M)_\rho \leq 0, \quad (1)$$

where

$$S_M = p(a_M b_M) + \sum_{k=2}^M [p(a_k b_{k-1}) + p(a_{k-1} b_k)] - p(a_1 b_1) - \sum_{k=2}^M [p(a_k) + p(b_k)], \quad (2)$$

and $(S_M)_\rho$ is the expectation value of S_M in the state ρ . Here, a_k (b_k) with $k = 1, \dots, M$ represents dichotomic observables, and $p(a_k b_k)$ is the joint probability of obtaining $a_k = b_k = 1$.

Assuming the same detection efficiency for every party and setting, i.e., $\eta_A = \eta_B = \eta$, the value of S_M becomes

$$\eta^2 (S_M)_\rho - \eta(1 - \eta) \sum_{k=2}^M [p_\rho(a_k) + p_\rho(b_k)], \quad (3)$$

where $p_\rho(a_k)$ is the expectation value of $p(a_k)$ in the state

ρ . Therefore, inequality (1) is violated when $\eta > \eta_{\text{crit}}$, with

$$\eta_{\text{crit}_M} = \frac{\sum_{k=2}^M [p_\rho(a_k) + p_\rho(b_k)]}{(S_M)_\rho + \sum_{k=2}^M [p_\rho(a_k) + p_\rho(b_k)]}. \quad (4)$$

Any generic two-qubit pure states $\rho = |\psi\rangle\langle\psi|$, can be written (in a suitable basis) as

$$|\psi\rangle = \alpha|00\rangle - \beta|11\rangle \quad (5)$$

with $\alpha, \beta \in \mathbb{R}$ and $\alpha^2 + \beta^2 = 1$.

Let us consider the following eigenstates:

$$|a_1\rangle = |b_1\rangle = \cos \frac{\theta}{2} |0\rangle + \sin \frac{\theta}{2} |1\rangle, \quad (6a)$$

$$|a_k\rangle = |b_k\rangle = |0\rangle, \text{ with } k = 2, \dots, M, \quad (6b)$$

and choose θ such that $\tan^2 \frac{\theta}{2} = \frac{\alpha}{\beta}$. Then, $p_\rho(a_1 b_1) = 0$ and the critical efficiency becomes

$$\eta_{\text{crit}_M} = \frac{2M - 2}{2M - 3 + 2 \cos^2 \frac{\theta}{2}}, \quad (7)$$

which, when θ tends to zero (i.e., when the state tends to a product state), tends to

$$\eta_{\text{crit}_M} = \frac{2M - 2}{2M - 1}. \quad (8)$$

The important point here is that this value is *smaller* than the minimum value of η_{crit_M} for maximally entangled states [21], namely,

$$\eta_{\text{crit}_M}^{\text{mes}} = \frac{2M - 2}{M - 1 + M \cos \left(\frac{\pi}{2M} \right)} \quad (9)$$

Moreover, for Bell inequalities of the form (1), the value in the right hand side of (8) is the minimum detection efficiency needed for any quantum state and choice of settings.

Proof: Inserting the assumption of independent errors, the critical efficiency of inequality (1) becomes

$$\eta_{\text{crit}_M} = \frac{\sum_{k=2}^M [p(a_k) + p(b_k)]}{p(a_M b_M) + \sum_{k=2}^M [p(a_k b_{k-1}) + p(a_{k-1} b_k)] - p(a_1 b_1)}. \quad (10)$$

Clearly,

$$0 \leq p_\psi(a_j b_k) \leq \min [p_\psi(a_j), p_\psi(b_k)], \quad (11)$$

and the lowest possible bound is obtained when $p_\psi(a_1 b_1) = 0$ and $p_\psi(a_j b_k) = p_\psi(a_j) = p_\psi(b_k)$ for j and k not both

equal to one. We obtain

$$\eta_{\text{crit}_M} \geq \frac{(2M - 2)p_\psi(a_1)}{(2M - 1)p_\psi(a_1)} = \frac{2M - 2}{2M - 1}, \quad (12)$$

which cannot be achieved exactly, but arbitrarily close with

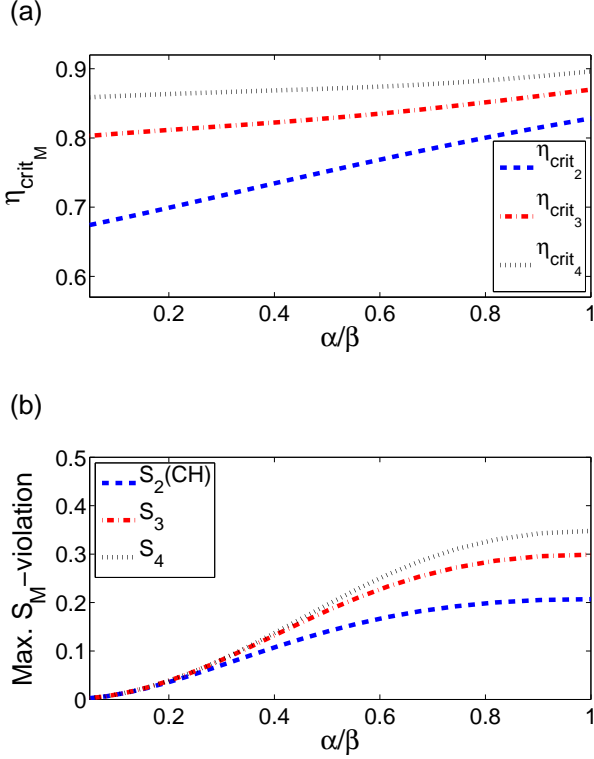


FIG. 1: (a) Minimum η_{crit_M} as a function of the degree of entanglement measured by α/β . (b) Maximum violation of $(S_M)_\rho \leq 0$ for a given α/β .

an appropriate quantum state, as shown by equation (7). ■

This proves that, in the absence of noise, almost product states give the best detection efficiency for chained Bell inequalities for any value of M .

We have also obtained the critical efficiency as a function the degree of entanglement measured by α/β and compared it with the corresponding maximal violation of the Bell inequality. By using the method of conjugate gradient, we have numerically found η_{crit_M} and the maximum values of S_M as a function of α/β . The results for $M = 2, 3, 4$ are shown in Fig. 1. The analytic expression for the state giving the largest violation for a given detection efficiency for the case $M = 2$ can be found in the Supplementary Material.

We observe that larger violations of S_M do not correspond to lower critical detection efficiencies. The value of S_M is not a good measure of nonlocality. For any number of settings, the lowest threshold efficiency occurs when the state is almost a product state rather than when it is a maximally entangled state. Indeed, as it is shown in the Supplementary Material, this conclusion also holds for less symmetrical Bell inequalities like the I_{3322} inequality [25–27].

Noise.—How does noise affect this conclusion? In the presence of white noise, the state becomes $\rho = (1 - q)|\psi\rangle\langle\psi| + \frac{q}{4}\mathbb{1}$ and the threshold detection for the chained Bell inequalities efficiency is changed to

$$\eta_{\text{crit}_M} = \frac{\sum_{k=2}^M [p_\rho(a_k) + p_\rho(b_k)] + \frac{q}{1-q}(M-1)}{(S_M)_\rho + \sum_{k=2}^M [p_\rho(a_k) + p_\rho(b_k)] + \frac{q}{2(1-q)}(M-1)}. \quad (13)$$

In Fig. 2 we show, for three different values of noise ($q = 0.01$, $q = 0.05$, and $q = 0.1$), the dependence of η_{crit_M} and the maximum values of S_2 and S_3 with the degree of entanglement of the initial pure state. We observe that, when the noise is different from 0, the best quantum state giving the lowest threshold is not an almost separable state, but a nonmaximally entangled state depending on q . However, the lower the noise q , the smaller the entanglement required to obtain the optimal threshold.

Furthermore, in Fig. 2(b) we observe that, the lower M is, the more resistant to noise is the violation of the Bell inequality. In fact, it is possible to calculate the maximum tolerated noise to violate the chained Bell inequalities. Given α/β and the maximal violation of S_M defined as $s_M^{\text{max}}(\alpha/\beta)$, the maximum tolerated noise is $q_{\text{max}} = \frac{2s_M^{\text{max}}(\alpha/\beta)}{2s_M^{\text{max}}(\alpha/\beta) + M - 1}$.

Using the method of conjugate gradient to minimize Eq. (13), it is also possible to obtain the threshold and the

required entanglement for any value of the noise q . The results are shown in Fig. 3. We observe that, for chained Bell inequalities, nonlocality and entanglement are simultaneously maximized *only in case of extreme noise*. A better threshold detection efficiency is obtained by lowering the noise and suitably decreasing the entanglement. From this we conclude that nonlocality and entanglement are synonymous only for extremely noisy scenarios.

Discussion.—In a general two-party M -setting Bell scenario (for any $M \geq 2$ finite), nonlocality and entanglement are, in the absence of noise, opposite properties in the following sense. We have argued that the critical detection efficiency η_{crit} is a good measure of nonlocality, since it marks the border where local hidden variable descriptions becomes possible: the smaller η_{crit} , the harder it is to express the joint probabilities with local models. Therefore, the result can be summarized by saying that, in a noiseless scenario, larger

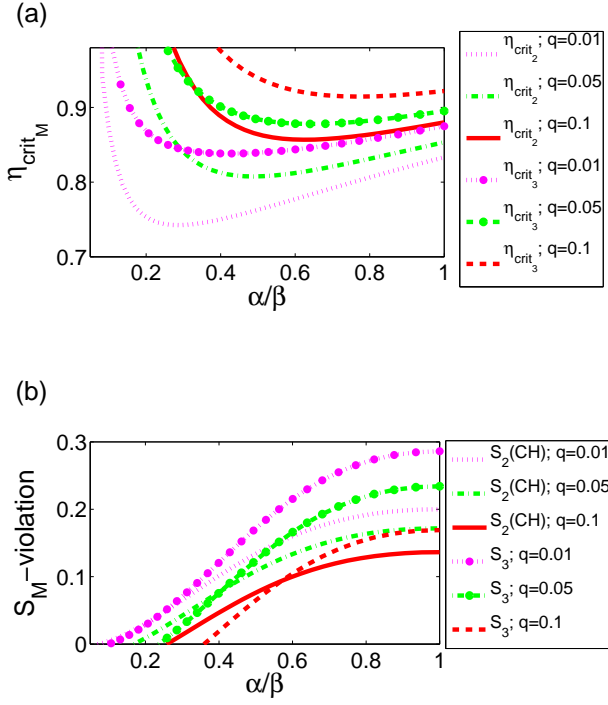


FIG. 2: (a) Values of η_{crit_M} and (b) maximum violation of the chained Bell inequality for different number of settings and different degree of noise (q).

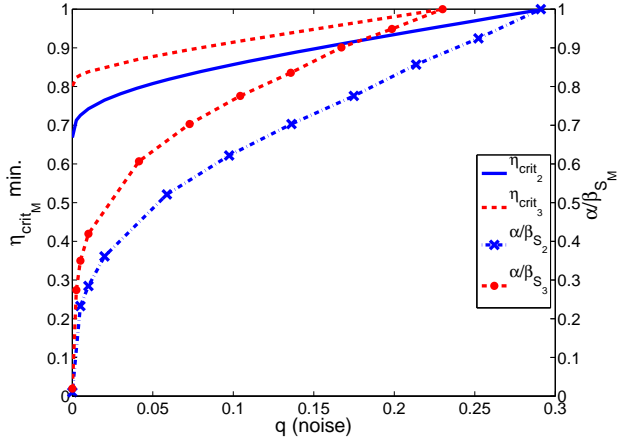


FIG. 3: Lowest threshold detection efficiency in the presence of noise for the S_2 and S_3 Bell inequalities. The value of α/β giving the best threshold is shown for each q .

nonlocality requires smaller entanglement; in the absence of noise, almost product states are the most nonlocal ones. When the noise is turned on, the most nonlocal states acquire some amount of entanglement; however, the smaller the noise is, the lower their entanglement becomes. This emphasizes that nonlocality and entanglement are not only different but, in many cases, opposite concepts.

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SUPPLEMENTARY MATERIAL

State giving the largest violation of S_2 for a given value of η

We use the projectors corresponding to the results a_2 and b_2 as the computational basis, so that using the projector

$$\Pi = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}. \quad (14)$$

and rotations from a_2 to a_1 and from b_2 to b_1 conveniently parameterized by

$$U_a = \begin{pmatrix} \sqrt{1-s} & \sqrt{s} \\ -\sqrt{s} & \sqrt{1-s} \end{pmatrix}, \quad U_b = \begin{pmatrix} \sqrt{1-t} & \sqrt{t} \\ -\sqrt{t} & \sqrt{1-t} \end{pmatrix}, \quad (15)$$

we obtain

$$\Pi_{a_2} = \Pi \otimes \mathbb{1}, \quad (16a)$$

$$\Pi_{b_2} = \mathbb{1} \otimes \Pi, \quad (16b)$$

$$\Pi_{a_2 b_2} = \Pi \otimes \Pi, \quad (16c)$$

$$\Pi_{a_2 b_1} = \Pi \otimes U_b^{-1} \Pi U_b, \quad (16d)$$

$$\Pi_{a_1 b_2} = U_a^{-1} \Pi U_a \otimes \Pi, \quad (16e)$$

$$\Pi_{a_1 b_1} = U_a^{-1} \Pi U_a \otimes U_b^{-1} \Pi U_b. \quad (16f)$$

Under the assumption of independent errors at equal rate, the CH operator including efficiency is

$$\beta = \eta^2 (\Pi_{a_2 b_2} + \Pi_{a_2 b_1} + \Pi_{a_1 b_2} - \Pi_{a_1 b_1}) - \eta (\Pi_{a_0} + \Pi_{b_0}) \quad (17)$$

Local realism bounds the expectation value of this measurement below zero, so any positive eigenvalue to the operator will give a violation. The eigenvalues are the solutions of the characteristic polynomial

$$\begin{aligned} & st\eta^5 [-st\eta^3 + (s+t)\eta(2\eta-1) - 3\eta + 2] \\ & + 2(\eta-1)\eta^3 [st(\eta^2 - \eta) - 1] \lambda \\ & - \eta^2(4\eta-5)\lambda^2 - 2\eta(\eta-2)\lambda^3 + \lambda^4 = 0. \end{aligned} \quad (18)$$

Seeking a maximum violation, we need to find the parameter values s and t that gives this maximum. Only the sum

$s+t$ and product st occur above, so we can also view this as finding values of $s+t$ and st that gives the maximum. The combination $s+t$ occurs only in the constant term in the polynomial (with a positive coefficient if $\eta > \frac{1}{2}$) so that, for a given value of st , the maximum highest solution is obtained when $s+t$ is minimal, i.e., when $s=t$. In this case,

$$\begin{aligned} p(\lambda) = & t^2\eta^5 [-t^2\eta^3 + 2t\eta(2\eta-1) - 3\eta + 2] \\ & + 2(\eta-1)\eta^3 [t^2(\eta^2 - \eta) - 1] \lambda \\ & - \eta^2(4\eta-5)\lambda^2 - 2\eta(\eta-2)\lambda^3 + \lambda^4 = 0. \end{aligned} \quad (19)$$

Changing to the Bell basis (in the order $|a_2 b_2\rangle \pm |a_2^\perp b_2^\perp\rangle, |a_2 b_2^\perp\rangle \pm |a_2^\perp b_2\rangle$ so that the singlet is last), the CH operator becomes

$$\begin{aligned} \beta = & \eta^2 \begin{pmatrix} 1-t & 1 & 0 & 0 \\ 1 & -2t^2+t+1 & 2t\sqrt{t}\sqrt{1-t} & 0 \\ 0 & 2t\sqrt{t}\sqrt{1-t} & 2t^2-t & 0 \\ 0 & 0 & 0 & t \end{pmatrix} \\ & - \eta \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \end{aligned} \quad (20)$$

and it is immediately clear that the singlet state is an eigenvector to this operator. We can read off the corresponding eigenvalue $\lambda_4 = \eta^2 t - \eta$, which is always negative. The remaining three eigenvalues can be obtained by solving the equation

$$\begin{aligned} & \frac{\lambda^3}{\eta^3} + [\eta(t-2) + 3] \frac{\lambda^2}{\eta^2} \\ & + [\eta^2(t^2 - 2t) + 2\eta(t-1) + 2] \frac{\lambda}{\eta} \\ & + \eta^3 t^3 - 3\eta^2 t^2 + 2\eta t^2 = 0. \end{aligned} \quad (21)$$

Solving with a general method (using complex numbers) will give the right answer. When the roots are real, the trigonometric method gives the following expression:

$$\begin{aligned} \lambda_1 = & -\frac{1}{3}\eta[3 + (-2+t)\eta] + \frac{2}{3}\eta\sqrt{3-6\eta+(4+2t-2t^2)\eta^2} \\ & \times \cos \left[\frac{1}{3} \arccos \left[\eta \left(9 - 18\eta + 8\eta^2 - 10t^3\eta^2 + 3t(3-6\eta+2\eta^2) \right. \right. \right. \\ & \left. \left. \left. - 3t^2(9-18\eta+4\eta^2) \right) \right] / \sqrt{(3-6\eta+2(t-2)(t+1)\eta^2)^3} \right], \end{aligned} \quad (22)$$

which is simpler than the variant suitable for complex roots. The other solutions can be obtained by adding $2\pi/3$ and

$4\pi/3$ to the arccos angle. These will be lower than λ_1 above.

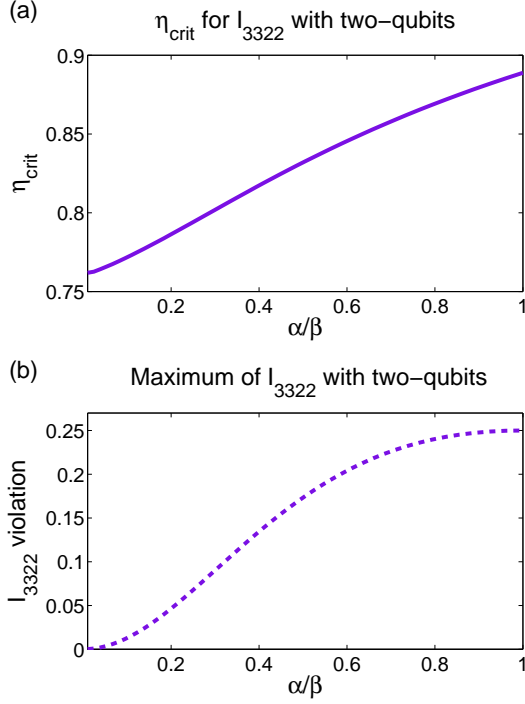


FIG. 4: (a) Minimum detection efficiency threshold and (b) maximum violation of the I_{3322} inequality as a function of the degree of entanglement.

The next step would be to differentiate λ_1 with respect to t and solve for 0, but that is not so easy. A simpler way is to use the equation as an implicit definition of λ as a function of t and do implicit differentiation, as in

$$\frac{d}{dt}p(\lambda(t)) = \lambda'(t)\frac{\partial}{\partial\lambda}p(\lambda) + \frac{\partial}{\partial t}p(\lambda) = 0 \quad (23)$$

Since we seek the maximum, $\lambda'(t) = 0$ and we arrive at

$$\lambda = \frac{\eta}{2(\eta-1)^2}(2t^2\eta^3 - 3t\eta(2\eta-1) + 3\eta-2). \quad (24)$$

Back substitution and elimination of some simple factors gives a fourth-degree polynomial equation for t ,

$$\begin{aligned} &4\eta^6 t^4 + 4\eta^4 (2\eta^2 - 10\eta + 5) t^3 \\ &+ \eta^2 (4\eta^4 - 48\eta^3 + 156\eta^2 - 132\eta + 33) t^2 \\ &+ 2(2\eta - 1)^2 (5\eta^2 - 16\eta + 8) t \\ &- (\eta - 2)(2\eta - 1)^2(3\eta - 2) = 0 \end{aligned} \quad (25)$$

This is solvable, but an analytic solution is very long. Reproducing that here is not very helpful; it is better to insert the efficiency η and solve for t to obtain the rotation between the two measurement setups. This can be used to obtain λ , and the eigenvectors are then simple to find.

Detection efficiency vs. entanglement in the I_{3322} inequality

The results in the main text shown that, for chained Bell inequalities of any number of settings, and in the absence of noise, product states are the most nonlocal ones. This result also holds for the I_{3322} inequality [25–27].

Fig. 4 shows the threshold detection efficiency and the violation of the I_{3322} inequality as a function of the degree of entanglement (measured by α/β). We observe that, in the absence of noise, almost product states are again those that require lower detection efficiencies. We also observe that the detection threshold for the I_{3322} is lower than the one for the S_3 inequality, which has the same number of local settings. For two-qubit systems, the maximum violation is $1/4$, and can be achieved with a maximally entangled state, as was previously shown in [27]. However, the minimum required efficiency with maximally entangled states is 0.8889, not 0.8284 as reported in N. Brunner and N. Gisin, Phys. Lett. A **372**, 3162 (2008). This value of 0.8889 can indeed be obtained analytically. For a maximally entangled state, it is easy to show that $\eta_{\text{crit}} \geq 2/(2.25)$.